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Uncertainty Analysis of Propellant Gauging System for Spacecraft

Amit Lal* and B. N. Raghunandan†
Indian Institute of Science, Bangalore 560 012, India

This paper discusses the traditional concept of propellant gauging system based on the gas law method for estimating the amount of residual propellant mass in spacecraft and the effects of the uncertainty in the instruments on the predicted amount of propellant. Because for accurate determination of the remaining mission life of the satellite it is extremely essential to estimate the amount of propellant present in the propellant tank of the spacecraft at various stages of its mission life, it is important to study the extent to which the uncertainty in various instruments and other measured parameters affect the predicted amount of propellant. With Monte Carlo simulation, it is found that the accuracy with which the propellant quantity can be estimated is highly sensitive to the precision of the propellant tank pressure sensor. It is also found that the uncertainties in the propellant tank pressure sensor and the pressurant tank temperature sensor result in uncertainty as well as overprediction of the amount of residual.

Nomenclature

$\mathrm{d}V_p/\mathrm{d}P_p$	=	pressurant tank stretch coefficients
$\mathrm{d}V_T/\mathrm{d}P_u$	=	propellant tank stretch coefficients
eV_L	=	the deviation of V_L from V_{L0}
$(P_p)_f$	=	pressurant tank pressure after repressurization
$(P_p)_i$	=	pressurant tank pressure before repressurization
$(P_u)_f$	=	propellant tank pressure after repressurization
$(P_u)_i$	=	propellant tank pressure before repressurization
S_{V_L}	=	standard deviation of the sample of propellant volumes
T_p	=	pressurant tank temperature
T_u	=	propellant tank temperature
V_L	=	estimated mean propellant volume present
$V_{L\max}$	=	upper uncertainty bound on volume of propellant present
$V_{L\min}$	=	lower uncertainty bound on volume of propellant present
V_{L0}	=	calculated propellant volume based on the measured values of all the parameters
V_p	=	unstressed pressurant tank volume
V_T	=	unstressed propellant tank volume
ΔP_p	=	pressurant tank pressure decrease due to repressurization
ΔP_u	=	propellant tank pressure increase due to repressurization

Introduction

ESTIMATION of the total mission life of a spacecraft is an important issue for the communication satellite industries. For accurate determination of the remaining mission life of the satellite it is essential to estimate the amount of propellant present in the propellant tank of the spacecraft at various stages of its mission life. Because the annual revenue incurred from a typical communication satellite operating at its full capacity is on the order of billion of dollars, premature removal of spacecraft from their orbits results in heavy losses.

Various techniques such as the bookkeeping method, the gas law method, numerical modeling techniques,[1] and use of capacitive sensors have been employed in the past for accurate determination of the amount of propellant present in a spacecraft. Their comparative advantages and disadvantages have also been studied.[2]

Chobotov and Purohit[3] suggested and developed a system based on gas law calculations to estimate the amount of propellant left in the propellant tank of a satellite. This paper discusses the effects of the uncertainty in the instruments employed in the propellant gauging system suggested by Chobotov and Purohit[3] on the predicted amount of propellant present in the propellant tank.

Description of the Model

Chobotov and Purohit[3] suggested an isothermal model for propellant gauging. The formulation of their model with the isothermal assumption and accounting for the tank stretch leads to the following equation for the amount of propellant present in the propellant tank[3]:

$$V_L = \left[V_T + (P_u)_f\left(\frac{\mathrm{d}V_T}{\mathrm{d}P_u}\right)\right] - \left[V_p + (P_p)_f\left(\frac{\mathrm{d}V_P}{\mathrm{d}P_p}\right)\right]\left(\frac{T_u}{T_p}\right)\left(\frac{\Delta P_p}{\Delta P_u}\right) \tag{1}$$

It has been pointed out by Chobotov and Purohit[3] that though Eq. (1) does not include real system effects such as pressurant gas compressibility, pressurant gas solubility in the propellant, and heat transfer effects, it is adequate for illustrative purposes.

Error Analysis

Equation (1) suggests that the precision with which the volume of the propellant present in the propellant tank is estimated depends on the precision of eight parameters, namely, pressurant tank volume and its stretch, propellant tank volume and its stretch, and the pressure and temperature sensors mounted on the pressurant and propellant tanks. To study how the calculated value of the volume of the propellant gets affected by the precision of an individual parameter independent of the precision of all other parameters, a particular parameter was chosen at a time and the precision of that particular parameter was varied, fixing the uncertainty in all other parameters at zero. As a result it was possible, using a Monte Carlo technique, to study the variation in the precision and the error in the calculated value of the volume of propellant present in the tank with the variation in the precision of the particular parameter chosen.

The sample set of readings used for the error analysis is given in Table 1. These sample data have been taken from Ref. 3.

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*Research Scholar, Department of Aerospace Engineering.
†Professor, Department of Aerospace Engineering.

Table 1 Sample set of readings for various parameters

Symbol	Values[a]	
V_T	19,151.4 in.^{3}	0.3138352 m^3
V_p	1,487.58 in.^{3}	0.02437707 m^3
T_u	300 K	300 K
T_p	300 K	300 K
$(P_u)_i$	212.71 psia	1,466,584 Pa
$(P_u)_f$	215.06 psia	1,482,787 Pa
$(P_p)_i$	241.03 psia	1,661,843 Pa
$(P_p)_f$	227.05 psia	1,565,455 Pa
$\left(\dfrac{dV_T}{dP_T}\right)$	0.5783 in.^{3}/psi	1.37447×10^{-9} m^3/Pa
$\left(\dfrac{dV_p}{dP_p}\right)$	0.02535 in.^{3}/psi	6.02504×10^{-11} m^3/Pa

[a]Taken from test hardware characteristics.[3]

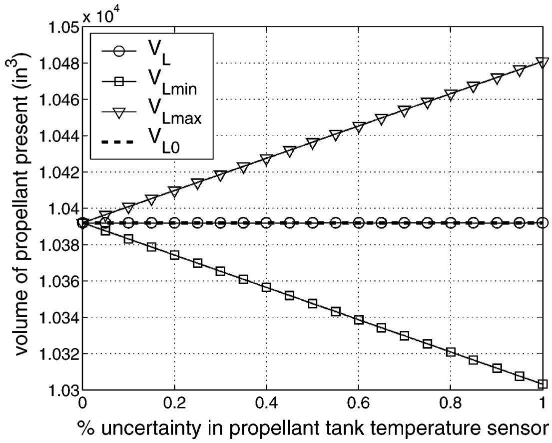

Fig. 1 Uncertainty bounds on V_L vs uncertainty in propellant tank temperature sensor.

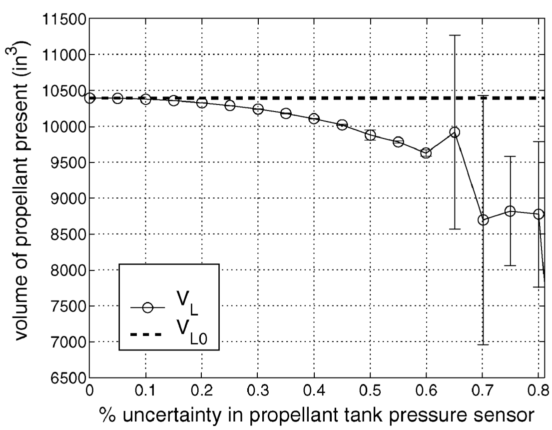

Fig. 2 Variation in V_L with uncertainty in propellant tank pressure sensor.

Implementation of Monte Carlo Method

This section illustrates the method of implementation of the Monte Carlo method used for error analysis by illustrating the effect of the uncertainty in the measurements obtained by the propellant tank temperature sensor on the propellant volume calculated.

Assuming that the true values of the ullage temperature in the propellant tank are normally distributed about its measured value[4] ($T_u = 300$ K), a set of 10^6 normally distributed random values ($T_{u1}, T_{u2}, \ldots, T_{un}$; $n = 10^6$) representing true values of ullage gas temperature were generated, with mean equal to the measured value ($T_u = 300$ K) and standard deviation corresponding to the precision of the temperature sensor under consideration. The uncertainty in the measured values of all other parameters is assumed to be zero.

For the set of 10^6 values ($T_{u1}, T_{u2}, \ldots, T_{un}$) representing true values of the temperature of the ullage gas and assuming all other measured parameters to have exact values, as given in Table 1, a set of 10^6 true values ($V_{L1}, V_{L2}, \ldots, V_{Ln}$) for the volume of propellant present in the propellant tank can be generated using Eq. (1) with each value V_{Li} corresponding to a particular value T_{ui}. The mean (V_L) and standard deviation (S_{V_L}), of the values ($V_{L1}, V_{L2}, \ldots, V_{Ln}$) represent the best estimate of the propellant volume present in the tank and its uncertainty, respectively.

Also, considering the value of temperature recorded by the temperature sensor ($T_u = 300$ K) to be the true value of the ullage gas temperature (as assumed for all other parameters) and using Eq. (1), an exact value for volume of propellant present in the tank (say V_{L0}) can be calculated. The difference $eV_L = |V_{L0} - V_L|$ represents the error which we will be making in our estimation of propellant volume present when using the measured value of the propellant tank temperature sensor, employing Eq. (1), and not considering the uncertainty in the measurement of the temperature sensor.

The effect of the uncertainty in all other parameters on the calculated volume of propellant present was studied in a similar way. All the computation was done by writing specific programs using the computational software MATLAB®.[5]

Results and Discussion

Figure 1 shows the quantitative variation in V_L, with the uncertainty bounds on it, as the uncertainty in the propellant tank temperature sensor is varied from 0 to 1% (3σ limits). The 0.13th and 99.87th percentile values of propellant volume ($V_{L1}, V_{L2}, \ldots, V_{Ln}$) have been used here for lower and upper confidence limits ($V_{L\min}$ and $V_{L\max}$), respectively, in consistent with the 99.74% confidence interval used in the two-tailed test (3σ limits) in inferential statistics.[6]

Two important observations can be made from Fig. 1. First, V_{L0} coincides with V_L; that is, the difference $eV_L = |V_{L0} - V_L|$ was always found to be less than $3S_{V_L}/\sqrt{n}$, which is the uncertainty with which V_L is determined. Second, the uncertainty in the determination of the propellant volume present is symmetrical on either side of V_L. This is due to the fact that the sample set of values obtained for propellant volumes ($V_{L1}, V_{L2}, \ldots, V_{Ln}$) are normally distributed about their mean value V_L. Thus, an uncertainty in the propellant tank temperature sensor does not introduce any error when the volume of propellant present in the tank is approximated by V_{L0}; it only introduces a certain degree of uncertainty in the determination of the volume of the propellant.

Similar investigations were made to study the effect of uncertainty in all other parameters individually on the estimated volume of propellant present in the propellant tank and it was found that they all, except the uncertainty in propellant tank pressure sensor, show trends qualitatively similar to that of the effect of the uncertainty in the propellant tank temperature sensor alone on the estimated volume of propellant present.

Figure 2 shows the variation of V_L as the uncertainty in propellant tank pressure sensor is varied. The vertical line segments in the plot represents the uncertainty bars with which V_L is determined, that is, $3S_{V_L}/\sqrt{n}$ on either side of V_L. A horizontal line corresponding to V_{L0} is also shown; the deviation of the plot of V_L from this line indicates the error eV_L. Figure 3 shows the quantitative variation in the volume of propellant present and the uncertainty bounds for smaller values of uncertainty in tank pressure sensor.

It can be noted from Fig. 2 that V_L, which is the best estimate of the propellant volume present, continues to decrease more and more from V_{L0} as the uncertainty in the propellant tank pressure sensor increases. It can be noted further that the error $eV_L = |V_{L0} - V_L|$ is beyond what can be attributed to the uncertainty in the determination of V_L. Thus, it can be concluded that the uncertainty in the propellant tank pressure sensor, unlike that in the rest of the parameters, does introduce an error when the volume of propellant present in the propellant tank is approximated by V_{L0}, besides introducing a certain degree of uncertainty into the determination of the volume of the propellant. Thus, if V_{L0} is used to quantify the propellant volume present, then it will be always be an overprediction.

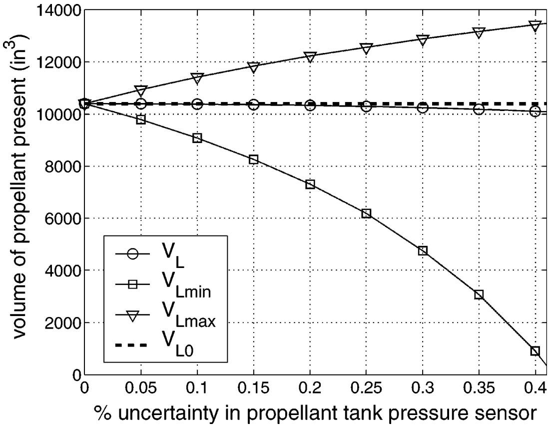

Fig. 3 Uncertainty bounds on V_L vs uncertainty in propellant tank pressure sensor.

The reason for this is the interesting fact that when a population, say P_Y, has a normal distribution about a mean value, say μ, then the new population, say P_Z, formed by taking the inverse of all the individual elements of the population P_Y has a frequency distribution that is positively skewed, with its mean lying on the higher side of $1/\mu$. Further, the shift of the mean of the population P_Z from $1/\mu$ increases with increased spread (i.e., decreased precision) of the population P_Y.

It can be seen that the parameter ΔP_u is appearing in the denominator of the second term of Eq. (1). Because the propellant tank pressure sensor has an uncertainty associated with it, the true values for both $(P_u)_i$ and $(P_u)_f$ will be normally distributed about their respective means. Consequently, the true value of the parameter ΔP_u is also normally distributed about its mean value, which is $(P_u)_f - (P_u)_i = 2.35$ psi. Following the discussion in the previous paragraph, the mean value of the second term of Eq. (1) is expected to increase as the uncertainty in the propellant tank pressure sensor increases; consequently decreasing the mean propellant volume, V_L. Note that the population of the true value of the propellant volume present is negatively skewed. Consequently, the uncertainty bound on the determination of the propellant volume present is unsymmetrical on either side of V_L (Fig. 3). This explains the reason for introduction of error when the propellant volume present is approximated by V_{L0}.

A further observation that can be made from Fig. 2 is that as the uncertainty in the propellant tank pressure sensor increases beyond about 0.6%, the estimate V_L becomes random and unpredictable. This is because the mean value of $\Delta P_u (= 2.35$ psi) lies very close to zero. So, as the uncertainty in the propellant tank pressure sensor increases, the probability of the random values of ΔP_u falling close to zero increases. This results in a large dispersion in the values of $V_{L1}, V_{L2}, \ldots, V_{Ln}$. Note that this increase in dispersion depends not only on the fraction of elements of ΔP_u that fall in the neighborhood of zero, but also on the extent to which they fall close to zero. A single element of ΔP_u having magnitude as small as, say, 10^{-6} can well affect the dispersion more than, say, 10 elements of ΔP_u having magnitude 0.1. This explains the increase in randomness and uncertainty in the estimated mean propellant volume V_L.

The study of the effect of uncertainty in the pressurant tank temperature sensor (T_p) on the estimated propellant volume did not show significant bias error in spite of appearing in the denominator of the second term of Eq. (1) because its value ($T_p = 300$ K) lies farther away from zero than that of ($\Delta P_u = 2.35$ psi).

Table 2 lists the sensitivity of the estimated mean propellant V_L with respect to the uncertainty in the individual parameters; that is, the ratio of uncertainty in V_L to uncertainty in the corresponding parameter. It can be noted that V_L is highly sensitive to the uncertainty in the propellant tank pressor sensor of Chobotov and Purohit.[3] This explains the need to use a pressure transducer having uncertainty as low as 0.042%.

A Monte Carlo simulation for studying the effects of uncertainty in all the parameters simultaneously on the calculated value of the propellant volume present was also done. Figures 4 and 5 show the results of the simulation for which the uncertainty in all the parameters was fixed at 1%, except that for the propellant tank pressure sensor, whose uncertainty was varied as before. Comparing Figs. 4 and 5 with Figs. 2 and 3, respectively, it can be noted that the plots are more or less similar, both qualitatively and quantitatively. The difference between Figs. 5 and 3 for small values of uncertainty in the propellant tank pressure sensor is due to the uncertainties in the other parameters considered in the former and not in the latter. Thus it can be concluded that the accuracy and uncertainty of the estimated propellant volume is mainly influenced by the uncertainty in the propellant tank pressure sensor alone. The effect of uncertainty in all other parameters on the estimated propellant volume is marginal.

All the above investigations were carried out when the propellant fill fraction of the tank was about 54% (calculated based on the unstressed propellant tank volume). Investigation was made, further, to study variation of the bias error in estimating the remaining propellant with a decrease in propellant fill fraction of the tank. Figure 6

Table 2 Sensitivity of V_L to uncertainty in the individual parameters

Parameter under consideration	Sensitivity[a]
Propellant tank pressure sensor	124.577
Pressurant tank pressure sensor	20.225
Propellant tank volume	1.8429
Pressurant tank volume	0.8516
Pressurant tank temperature sensor	0.854
Propellant tank temperature sensor	0.854
Pressurant tank stretch	0.033
Propellant tank stretch	0.012

[a]Ratio of uncertainty in V_L to uncertainty in the corresponding parameter.

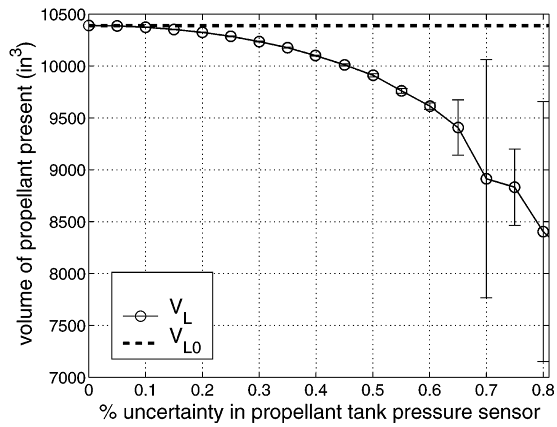

Fig. 4 Variation in V_L with uncertainty in propellant tank pressure sensor; uncertainty in all other parameters fixed at 1%.

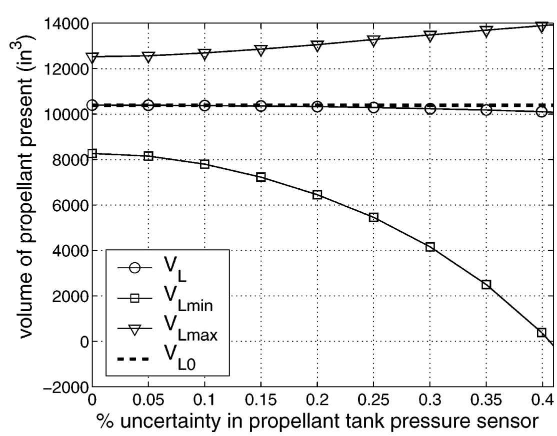

Fig. 5 Uncertainty bounds on V_L vs uncertainty in propellant tank pressure sensor; uncertainty in all other parameters fixed at 1%.

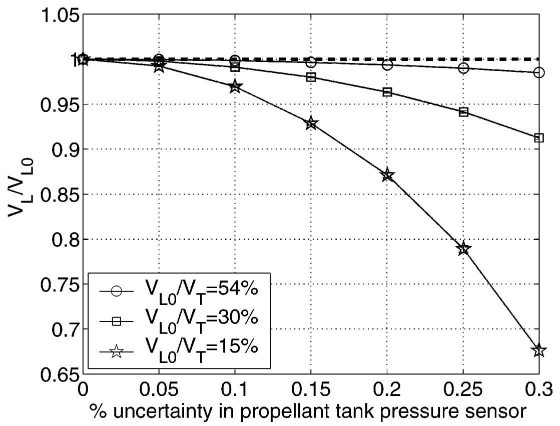

Fig. 6 Variation of bias error for different propellant fill fractions.

shows the deviation of V_L from V_{L0} as the uncertainty in the propellant tank pressure sensor increases, for three different propellant fill fractions: 54%, 30%, and 15%. For ease of comparison, the propellant volume has been normalized by dividing by the calculated propellant volume V_{L0} at each stage. It can be observed from Fig. 6 that the normalized error, $eV_L/V_{L0} = |V_{L0} - V_L|/V_{L0}$, increases as the propellant fill fraction in the tank decreases.

To obtain the sample set of values for this study it is assumed that after repressurization $P_{uf} = P_{pf} - 11.99$, and then the values of ΔP_p and ΔP_u for the three propellant fill fractions were calculated, assuming that the total mass of the pressurizing gas in the two tanks is conserved. Basically, this is assuming that we always repressurize the propellant tank up to 11.99 psi of the pressurant tank (as for the case given in Table 1).

Thus, it can be concluded that the percentage error incurred when the calculated propellant volume V_{L0} is used to approximate the amount of propellant present in the tank, instead of using the the estimated mean propellant volume V_L, keeps on increasing more and more as the propellant fill fraction in the tank decreases, and the error eV_L grows too large to be neglected from practical consideration.

Conclusions

Error analysis of the isothermal model of the propellant gauging system using the Monte Carlo method has revealed many interesting facts. In the light of the results obtained, we can conclude that the principal part of the uncertainty in the estimated propellant volume V_L is due to the uncertainty in the propellant tank pressure sensor. The effect of uncertainty in other parameters literally gets overshadowed by the uncertainty in the propellant tank pressure sensor.

The other major conclusions are as follows:

1) The uncertainty bound on the estimated propellant volume present is unsymmetrical due to the effect of uncertainty in the propellant tank pressure sensor.

2) The propellant volume calculated using the measured values of the various parameters tend to be an overestimate. The degree to which it overpredicts increases with the increase in the uncertainty in the propellant tank pressure sensor.

3) The percentage error incurred when the calculated propellant volume V_{L0} is used to approximate the amount of propellant present in the tank, instead of using the estimated mean propellant volume V_L, keeps on increasing more and more as the propellant fill fraction in the tank decreases.

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